



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 (a) By differentiating e^{-x^2} , find the Maclaurin's series for e^{-x^2} up to and including the term in x^2 . [5]

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(b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its lowest terms. [2]

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2 The variables x and y are related by the differential equation

$$9 \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + y = 3x^2 + 30x.$$

(a) Find the general solution for y in terms of x . [6]

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(b) State an approximate solution for large positive values of x . [1]

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3 (a) Show that the system of equations

$$\begin{aligned} x - 2y - 4z &= 1, \\ x - 2y + kz &= 1, \\ -x + 2y + 2z &= 1, \end{aligned}$$

where k is a constant, does not have a unique solution. [2]

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(b) Given that $k = -4$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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- (c) Given instead that $k = -2$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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- (d) For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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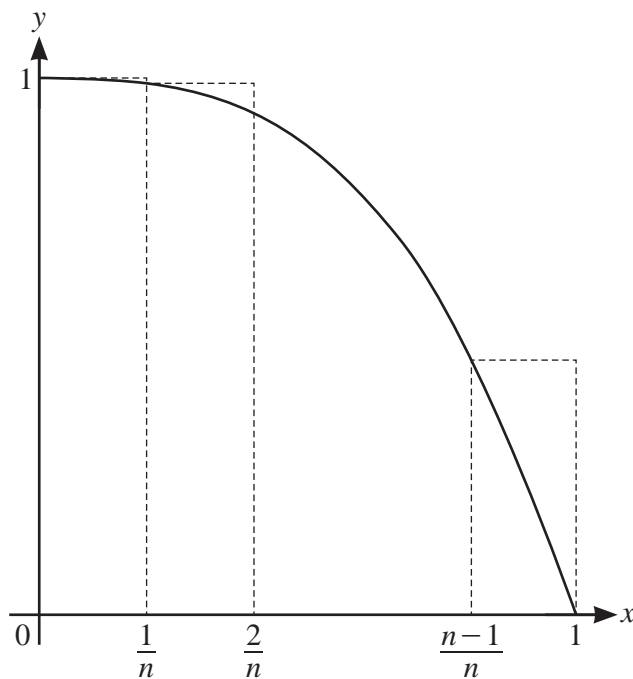
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The diagram shows the curve with equation $y = 1 - x^3$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1 - x^3) dx \leq \frac{3n^2 + 2n - 1}{4n^2}. \quad [4]$$

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 (b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^3) dx$. [4]

5 It is given that

$$x = \sinh^{-1}t, \quad y = \cos^{-1}t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t , show that $\frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}}$. [4]

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(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer. [5]

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7 The matrix \mathbf{P} is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of \mathbf{P} . [1]

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(b) Use the characteristic equation of \mathbf{P} to find \mathbf{P}^{-1} . [4]

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The 3×3 matrix \mathbf{A} has distinct eigenvalues $b, -1, 1$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

respectively.

- (c) Find \mathbf{A} in terms of b . [4]

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8 (a) Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

(b) Starting from the definitions of \coth and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1. \quad [3]$$

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